

OKLAHOMA STATE UNIVERSITY  
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING



**ECEN 4503**  
**Random Signals and Noise**  
**Spring 2007**



**Midterm Exam #2**

**For all students, choose any four out of five problems.**  
*Please specify which four listed below to be graded*

1) \_\_\_\_\_; 2) \_\_\_\_\_; 3) \_\_\_\_\_; 4) \_\_\_\_\_;  
:

**Name :** \_\_\_\_\_

**E-Mail Address:** \_\_\_\_\_

**Problem 1:**

A random variable  $X$  is uniformly distributed on  $(0, 6)$ . If  $X$  is transformed to a new random variable  $Y = 2(X - 3)^2 - 4$ , find a) the probability density function of  $Y$  and b)  $\bar{Y}$ .

**Problem 2:**

Find a value of the constant  $b$  so that the function

$$f_{x,y}(x, y) = bxy^2 \exp(-2xy)u(x-2)u(y-1)$$

is a valid joint density function.

**Problem 3:**

The random variables  $X$  and  $Y$  are statistically independent with exponential densities

$$f_X(x) = \alpha e^{-\alpha x} u(x), \text{ and}$$

$$f_Y(y) = \beta e^{-\beta y} u(y).$$

Find the probability density function of the random variable  $W = \max(X, Y)$ .

**Problem 4:**

Two random variables  $X$  and  $Y$  are defined by  $\bar{X} = 0$ ,  $\bar{Y} = -1$ ,  $\overline{X^2} = 2$ ,  $\overline{Y^2} = 4$  and  $R_{XY} = -2$ .

Two new random variables  $U$  and  $V$  are defined as

$$U = 2X + Y$$

$$V = -X - 3Y$$

Find  $\bar{U}$ ,  $\bar{V}$ ,  $R_{UV}$ , and  $\sigma_U^2$ .

**Problem 5:**

Zero-mean Gaussian random variables  $X_1$ ,  $X_2$ , and  $X_3$  having a covariance matrix

$$[C_x] = \begin{bmatrix} 4 & 2.05 & 1.05 \\ 2.05 & 4 & 2.05 \\ 1.05 & 2.05 & 4 \end{bmatrix}$$

are transformed to new random variables

$$Y_1 = 5X_1 + 2X_2 - X_3$$

$$Y_2 = -X_1 + 3X_2 + X_3 .$$

$$Y_3 = 2X_1 - X_2 + 2X_3$$

Find the covariance matrix of  $Y_1$ ,  $Y_2$ , and  $Y_3$  .